REPORT 1

A METHOD OF PREDICTING THE OPTIMUM LUNAR LANDING AREA FOR A MANNED SPACECRAFT

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SUMMARY

From a safety aspect, our present knowledge of the moon is inadequate to make a fast decision concerning the best site for landing a manned spacecraft. To date, ten sites have been proposed from which we must select the safest. Additional data from the areas selected are necessary in order to compare the results with data obtained from terrestrial samples of the site where Surveyor has landed. This paper proposes a method of successive transformations of observational data obtained during the Surveyor program from which we can expect to acquire a considerable amount of information regarding the proposed sites.

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1. INTRODUCTION

From a safety aspect, our present knowledge of the moon is inadequate to make a fast decision concerning the best site for landing a manned spacecraft. To date, ten sites have been proposed from which we must select the safest. Therefore, more lunar spectral-photometric, radiometric, and polarimetric observations are necessary to define the nature of the ground and, more precisely, the material composing the lunar surface. In other words, we need additional data from the areas selected in order to compare the results with data obtained from terrestrial samples.

At the same time, it would be useful to compare those areas proposed with respect to the site where Surveyor has landed. By extrapolating from the Surveyor data, we can expect to acquire a considerable amount of information about the other sites. The major problem, of course, is the method of extrapolation.

Direct comparison between the characteristics of areas observed and those of the reference source is not advisable if the aim of the research is to look at the landing safety aspects in all sites. In effect, with such a direct comparison, the discontinuities of the ground between the areas observed and the reference source would be neglected. In the same way, the observation of a few intermediate points does not help too much because we cannot satisfactorily discriminate between variations in the nature of the ground represented by such points. With a method such as the successive transformations of observational data, however, the extrapolation would not be difficult and it could be applied to any data reduction problem.

2. PROPOSED METHOD OF EXTRAPOLATION

As an example, let (C_i) represent the color obtained from observations using the infrared filter; let $(C_i)_o$ represent the color corresponding to the point A and $(C_i)_1$, $(C_i)_2$, $(C_i)_3$, $(C_i)_4$ and $(C_i)_5$ be the colors corresponding to points 1, 2, 3, 4, 5 for the same longitude but above A. Let $(C_i'')_1$, $(C_i'')_2$, $(C_i'')_3$, $(C_i'')_4$ and $(C_i'')_5$ represent those which are at the same longitude but below A. In the same manner, let $(C_i')_1$, $(C_i')_2$, $(C_i')_3$, $(C_i')_4$ and $(C_i')_5$ represent the points which are situated at the same latitude as A.

Using the color of A as a reference, we have the following relationships:

$$\frac{(C_i)_o}{(C_i)_1} = a \; ; \; \frac{(C_i)_o}{(C_i')_1} = a' \; ; \; \frac{(C_i)_o}{(C_i'')_1} = a''$$

$$\frac{(C_i)_o}{(C_i)_2} = b \; ; \; \frac{(C_i)_o}{(C_i')_2} = b' \; ; \; \frac{(C_i)_o}{(C_i'')_2} = b''$$

$$\frac{(C_i)_o}{(C_i)_3} = c \; ; \; \frac{(C_i)_o}{(C_i')_3} = c' \; ; \; \frac{(C_i)_o}{(C_i'')_3} = c''$$

$$\frac{(C_i)_o}{(C_i)_4} = d \; ; \; \frac{(C_i)_o}{(C_i')_4} = d' \; ; \; \frac{(C_i)_o}{(C_i'')_4} = d''$$

$$\frac{(C_i)_o}{(C_i)_5} = e \; ; \; \frac{(C_i)_o}{(C_i')_5} = e' \; ; \; \frac{(C_i)_o}{(C_i'')_5} = e''$$

In other words, as is shown by Scheme No. 2 (see Appendix), we enlarge the area of A in order to acquire additional data to serve as reference for the extrapolations which are shown by Scheme No. 1 (see Appendix). For better results, it is necessary to use the ratios a, b, c, d, e, a', b', c', d', e', and a'', b'', c'', d'', e'' as compared to direct use of the colors. This method is preferable because an extrapolation is not as accurate as an interpolation.

From these relationships, we have:

$$\left.\begin{aligned} a(C_i)_1 - 2b(C_i)_2 - c(C_i)_3 + d(C_i)_4 + e(C_i)_5 &= 0 \\ a'(C_i')_1 - 2b'(C_i')_2 - c'(C_i')_3 + d'(C_i')_4 + e'(C_i')_5 &= 0 \\ a''(C_i'')_1 - 2b''(C_i'')_2 - c''(C_i'')_3 + d''(C_i'')_4 + e''(C_i'')_5 &= 0 \end{aligned}\right\} \qquad (1)$$

So, from one circle to another, we have

$$\begin{aligned} &\left[a(C_i)_1 - 2a'(C_i')_1 + a''(C_i'')_1 \right] \\ - &\left[2b(C_i)_2 - 4b'(C_i')_2 + 2b''(C_i'')_2 \right] \\ - &\left[c(C_i)_3 - 2c'(C_i')_3 + c''(C_i'')_3 \right] \\ + &\left[d(C_i)_4 + d''(C_i'')_4 \right] \\ + &\left[e(C_i)_5 - 2e'(C_i')_5 + e''(C_i'')_5 \right] = 0 \end{aligned} \qquad (2)$$

Then, from the relationships among $(C_i)_1$, $(C_i')_1$, $(C_i'')_1$, we obtain the following for the first circle:

$$a = \frac{a'(1 + a')}{a''} + \frac{a'' + 1}{2} \;;\; a' = \frac{1}{2}(a - 1) - 2a'' \text{ and}$$

$$a'' = -\frac{1 + 2a'}{4}$$

which reduces to:

$$(a - 1) - 2a' - 4a'' = 0$$

Next, if we replace the ratios by their respectives values, we obtain the following identity:

$$(c_i')_1(c_i'')_1 - 2(c_i)_1(c_i')_1 - 4(c_i)_1(c_i'')_1 \equiv \frac{(c_i)_1(c_i')_1(c_i'')_1}{(c_i)_o}$$

or:

$$\frac{(c_i)_o}{(c_i)_1} = 1 = a - 1 \equiv (c_i')_1(c_i'')_1$$

$$\frac{(c_i)_o}{(c_i')_1} = a' \equiv (c_i)_1(c_i')_1$$

$$\frac{(c_i)_o}{(c_i'')_1} = a'' \equiv (c_i)_1(c_i'')_1$$

$$\frac{(c_i)_1(c_i')_1(c_i'')_1}{(c_i)_o} \equiv 0$$

The next consideration is any point a_x between a and a' or any point a_y between a' and a''. The relationships between a_x and a, a' or between a_y and a', a'' must be established in the following manner:

(1) Theoretically, there is no variation of color with respect to the points belonging to a same longitude. However, there are small variations of color with longitude but they are due only to the variations of the ground's nature.

Accordingly, $a_x \approx a_m$

(2) The color varies with the latitude. For this reason, when enlarging area A in order to obtain a larger quantity of data for reference, we must not proceed too far from A, otherwise, we will accrue systematic errors in our evaluation of the ratios a, b, c, d, e; a', b', c', d', e' and a'', b'', c'', d'', e''.

Under the circumstances mentioned in steps (1) and (2), we can establish a correlation between the ratio $(C_i')_1/(C_i)_1$ of a, a' and the ratio $(C_i^n)_1/(C_i^x)_1$ of any point located between a and a'. The same consideration is applicable to the point a_y with respect to a' and a''. Therefore, $a_x = \frac{aa_n}{a'}$ and $a_y = \frac{a''a_o}{a'}$

From equation (2):

$$\left.\begin{aligned}
a &= \frac{a'(1 + a')}{a''} + \frac{a'' + 1}{2} ; \; a' = \frac{1}{2}(a - 1) - 2a'' ; \\
a'' &= -\frac{1 + 2a'}{4} \quad \text{for the 1st circle} \\
b &= \frac{b'(1 + b')}{b''} + \frac{b'' + 1}{2} ; \; b' = \frac{1}{2}(b - 1) - 2b'' ; \\
b'' &= -\frac{1 + 2b'}{4} \quad \text{for the 2nd circle} \\
c &= \frac{c'(1 + c')}{c''} + \frac{c'' + 1}{2} ; \; c' = \frac{1}{2}(c - 1) - 2c'' ; \\
c'' &= -\frac{1 + 2c'}{4} \quad \text{for the 3rd circle} \\
d &= \frac{d'(1 + d')}{d''} + \frac{d'' + 1}{2} ; \; d' = \frac{1}{2}(d - 1) - 2d'' ; \\
d'' &= -\frac{1 + 2d'}{4} \quad \text{for the 4th circle}
\end{aligned}\right\} \quad (3)$$

(Continued)

$$\left.\begin{array}{l} e = \dfrac{e'(1 + e')}{e''} + \dfrac{e'' + 1}{2} \ ; \ e' = \dfrac{1}{2}(e - 1) - 2e'' \ ; \\ e'' = - \dfrac{1 + 2e'}{4} \quad \text{for the 5th circle} \end{array}\right\} \quad (3)$$

It is evident then that the ratios $a, a', a''; b, b', b''; \ldots ; e, e', e''$ correspond to the points which are chosen (when observing the Moon) for analysing region A going from its center a_o to the limit e_x. The ratios $(a_x, a_y), (b_x, b_y), \ldots, (e_x, e_y)$ are the intermediate points for studying the lunar surface with respect to those other ratios serving as reference.

The parameters a_n and a_o are very important due to the fact that they are useful for the constant verification of the values obtained from observations of $(a_x, a_y), (b_x, b_y), \ldots, (e_x, e_y)$. For each circle of equation (3), these parameters are:

$$\left.\begin{array}{ll} a_x = \dfrac{a\, a_n}{a'} \ ; \ a_y = \dfrac{a''a_o}{a'} & \text{for the 1st circle} \\ b_x = \dfrac{b\, b_n}{b'} \ ; \ b_y = \dfrac{b''b_o}{b'} & \text{for the 2nd circle} \\ c_x = \dfrac{c\, c_n}{c'} \ ; \ c_y = \dfrac{c''c_o}{c'} & \text{for the 3rd circle} \\ d_x = \dfrac{d\, d_n}{d'} \ ; \ d_y = \dfrac{d''d_o}{d'} & \text{for the 4th circle} \\ e_x = \dfrac{e\, e_n}{e'} \ ; \ e_y = \dfrac{e''e_o}{e'} & \text{for the 5th circle} \end{array}\right\} \quad (4)$$

Because of the importance of a_n and a_o, we must proceed, when observing the Moon, in the following way:

(1) Obtain the colors for the points serving as reference and then compute the corresponding ratios with respect to the color of point A.

(2) Complete a similar step for the intermediate points (a_x, a_y), (b_x, b_y),..., (e_x, e_y); then derive the corresponding values (a_{n_x}, a_{y_x}) for proceeding to the verification cited by using equation (4).

Actually, equation (4) is a continuous transformation of data for any point considered with respect to the color of point A. In effect, when establishing the ratios a, b, c, d, e; a', b', c', d', e' and a'', b'', c'', d'', e'', we are transforming (C_i), $C'_i)$, (C''_i) with respect to $(C_i)_o$ in order to obtain the index of the color variation for inferring the physical properties of the new points which are compared with A. Also, for points a_x, b_x, c_x, d_x, e_x and a_y, b_y, c_y, d_y, e_y, we are doing a second transformation with respect to A and a, a', a'',...; e, e', e''; etc. Finally, with the points (a_{n_x}, a_{m_x}) and (a_{o_x}, a_{n_x}), we are doing a third transformation with respect to A; a, a', a'',...; e, e', e''; etc., and (a_x, a_y),..., (e_x, e_y). Because $a_x \approx a_m$ and $a_y \approx a_n$, the third transformations a_n and a_o are sufficient to give us the precise information which is desired about the physical properties for the points considered other than A. This is the physical meaning of the identities that we have seen before.

Now, consider in a given circle of region A, the different points a_x and a_y. For example, consider $(a_{x=1}, a_{x=2}, a_{x=3})_1$ and $(a_{y=1}, a_{y=2}, a_{y=3})_1$, which are some of the points situated between a, a' and a', a'' in the first circle. In this case, we have:

$$(a_{x=1} + a_{x=2} + a_{x=3})_1 = \frac{a}{a'} (a_{n_{x=1}} + a_{n_{x=2}} + a_{n_{x=3}})_1$$

$$(a_{y=1} + a_{y=2} + a_{y=3})_1 = \frac{a''}{a'} (a_{o=1} + a_{o=2} + a_{o=3})_1$$

Consequently, for all points X and Y situated in the first circle between a, a' and a', a'', we can express the following:

$$
\left.\begin{aligned}
&(a_{x=1} + a_{x=2} + a_{x=3} + \dots + a_{x=x})_1 \\
&= \left(\frac{a}{a'}\right)(a_{n_{x=1}} + a_{n_{x=2}} + a_{n_{x=3}} + \dots + a_{x=x}) = \left(\frac{a}{a'}\right)\sum_{x=a_o}^{x=x=a'} a_{n_x} \\
&(a_{y=1} + a_{y=2} + a_{y=3} + \dots + a_{y=Y})_1 \\
&= \left(\frac{a''}{a'}\right)(a_{o=1} + a_{o=2} + a_{o=3} + \dots + a_{o=Y}) = \left(\frac{a''}{a'}\right)\sum_{x=a_o}^{x=Y=a'} a_{o_y}
\end{aligned}\right\} \quad (5)
$$

In going from one circle to another, we obtain the following for region A:

$$
\left.\begin{aligned}
&\left(\frac{a}{a'}\right)\sum_{x=a_o}^{x=a'} a_{n_x} = \alpha P_1 \quad ; \quad \left(\frac{a''}{a'}\right)\sum_{x=a_o}^{s=a'} a_{o_Y} = \alpha' Q_1 \\
&\left(\frac{b}{b'}\right)\sum_{x=a_o}^{x=b'} b_{n_x} = \beta P_2 \quad ; \quad \left(\frac{b''}{b'}\right)\sum_{x=a_o}^{x=b'} b_{o_Y} = \beta' Q_2 \\
&\left(\frac{c}{c'}\right)\sum_{x=a_o}^{x=c'} c_{n_x} = \gamma P_3 \quad ; \quad \left(\frac{c''}{c'}\right)\sum_{x=a_o}^{x=c'} c_{o_Y} = \gamma' Q_3 \\
&\left(\frac{d}{d'}\right)\sum_{x=a_o}^{x=d'} d_{n_x} = \delta P_4 \quad ; \quad \left(\frac{d''}{d'}\right)\sum_{x=a_o}^{x=d'} d_{o_Y} = \delta' Q_4
\end{aligned}\right\} \quad (6)
$$

(Continued)

$$\left.\begin{array}{l} \left(\frac{e}{,}\right)_{e} \sum_{x=a_o}^{x=e'} e_{n_x} = \epsilon P_5 \quad ; \quad \left(\frac{e}{,}\right)^{''}_{e} \sum_{x=a_o}^{x=e'} e_{o_Y} = \epsilon' Q_5 \end{array}\right\} \quad (6)$$

Where: α, β, γ, δ, ϵ, and α', β', γ', δ', ϵ', are the transformation coefficients, respectively, of the points P_1, P_2, P_3, P_4, P_5 and Q_1, Q_2, Q_3, Q_4, Q_5. These points represent the observational data which must be successively transformed with respect to P_o, i.e., the point where Surveyor landed. Thus, if such points are the source of observational data that we constantly compare with the physical parameters of P_o, then the transformation coefficients tell us how these physical parameters are varying with location from P_o to other points on the lunar surface.

In this last instance, it should be noted that equation (6) is good for studying any kind of data we need with respect to comparable data related to P_o. In effect, it would be sufficient to define (P_1, Q_1), (P_2, Q_2),..., according to our needs for comparing subsequent points with the reference point P_o.

For example, let $P_o = 0.18$, $P_1 = 0.23$, $P_2 = 0.21$, $P_3 = 0.14$, $P_4 = 0.09$ and $P_5 = 0.17$ be arbitrary values to illustrate the subsequent computations. Let N be the number of successive transformations done for any point P with respect to P_o. If we consider the circles of the region A, then we have:

$$\left.\begin{array}{l} \text{For the 1st circle:} \\ P_o = \alpha P_1 + \frac{N}{100} \text{ and the error } \Delta P = (\alpha P_1 + \frac{N}{100} - P_o \\ \text{For the 2nd circle:} \\ P_o = \beta P_2 + \frac{N}{100} \text{ and the error } \Delta P = (\beta P_2 + \frac{N}{100} - P_o \\ \text{For the 3rd circle:} \\ P_o = \gamma P_3 + \frac{N}{100} \text{ and the error } \Delta P = (\gamma P_3 + \frac{N}{100} - P_o \end{array}\right\} \quad (7)$$

(Continued)

For the 4th circle:

$$P_o = \delta P_4 + \frac{N}{100} \quad \text{and the error} \quad \Delta P = (\delta P_4 + \frac{N}{100}) - P_o$$

For the 5th circle:

$$P_o = \epsilon P_5 + \frac{N}{100} \quad \text{and the error} \quad \Delta P = (\epsilon P_5 + \frac{N}{100}) - P_o \qquad (7)$$

In these examples, all computations are carried out and shown in the final pages of this same section. From the equations listed above, however, we can see that errors must be on the same order of magnitude. This is due to the fact that, when transforming data from a given P to P_o, we are not going directly from one to another but successively passing through the series of points situated between them. Also, as is shown by Scheme 4 (see Appendix), the variation of observational data with respect to P_o is obtained with a high degree of accuracy because the correlations a/a', b/b',..., e/e' are constantly providing the exact location of points whose data is transformed.

It is not practical to proceed past area A because the required matrix would be excessively long and involved. Consequently, it is desirable to use an equation capable of inducing data within the same error $(-0.02 < \Delta P < 0.02)$ shown in developing the previous five examples. Accordingly, consider the last number of example No. 5:

$$\frac{P_5}{P_o}\left[\left(\frac{P_1}{P_4}\right)^5 \cdot \frac{(P_3)^7}{(P_2)^{10}}\right]$$

At first, we see that the number N of successive transformation is a function of the number of points considered in the series $P_o, P_1, \ldots, P_n$ while the transformation coefficient is a function of P itself. For this reason, the last number of the example mentioned is composed as follows:

(1) The ratio P_5/P_o corresponding to the extremities of the series

(2) A product of ratios whose number is depending on N

(3) Each such ratio is a new series, smaller than that which precedes it

(4) The powers simulataneously depend on N and P.

From this analysis of the last number of example No. 5, it is then possible to define the final equation with the same shape as equation (7), proceeding in the following way:

(1) We establish the condition "sine quanon" that points considered be of equal distance between them. Then N=0 for P_o, N=1 for P_1, N=2 for P_2 and so on if the distances $P_o \longrightarrow P_1 = P_1 \longrightarrow P_2 = P_2 \longrightarrow P_3, \ldots$

(2) To avoid systematic errors in the reduction of data to the maximum extent, do not take distances greater than 2° in the lunar coordinates when observing.

(3) We determine a, b, c, d, e ; a', b', c', d', e' and a'', b'', c'', d'', e'' from the observational data in order to define the P_n and Q_n.

(4) We adopt η_p, η_a as the summation of the successive transformation coefficients $(\alpha, \beta, \gamma, \ldots)_{P_o \rightarrow P}$ and $(\alpha', \beta', \gamma', \ldots)_{a_o \rightarrow a}$. So for any P, for example, we can now write as follows:

First Case

$$\left.\begin{array}{l} P_o = \eta P + \frac{N}{100} \quad \text{and the error is} \quad \Delta P = (\eta P + \frac{N}{100}) - P_o \\ \qquad \text{when} \quad \frac{N}{100} < P_o \\ \text{Second Case} \\ P_o = 2(\frac{N}{100} - \frac{2}{100}) - \eta P = 0.02\ (N - 2) - \eta P \\ \qquad \text{and the error is} \quad \Delta P = \left[0.02(N-2) - \eta P\right] - P_o \\ \qquad \text{when} \quad \frac{N}{100} \geqq P_o \end{array}\right\} \quad (8)$$

We have checked these equations by developing successive transformations of 45 examples, including the five examples previously cited. The results are given in Table I (see Appendix). Since the points were 46 with P_o when doing the successive transformations, then the enumeration N on Table I corresponds to the subsequent computations.

For the lunar observations done with more than a filter, we make the same computations after having defined the P, Q which correspond to the different colors observed. In the same way, if we desire to study other data obtained from the moon by means different than that of the Photometry, we first select the lunar coordinates which correspond to our N and proceed to the subsequent definitions of P, Q according to the nature of the physical parameters obtained by such other means.

Concerning the analysis of observational data after their reduction within the successive transformation method, the following work must be accomplished: NASA requests the study of fourteen areas from which the safest for landing a manned spacecraft must be selected. To recognize the safest among the many areas to be considered, the following method is used:

(1) The sites considered would be included in the band covered by Scheme 1 so we proceed to observe, with respect to the P_o of Surveyor, the whole band at N=1, N=2, N=3 ... until we reach an N beyond the last of the areas proposed. This is for the purpose of satisfying the condition which requires that the points being observed must be equally spaced. For avoiding, at best, the systematic errors, we would adopt a distance among the points not exceeding 2°.

(2) For each filter used, a, a', a''; b, b', b''; ... ; e, e', e'' are defined and the corresponding correlations are computed by using equations (3) and (4).

(3) The quantities P or Q are determined, the point observed being above or below the longitude of P_o, and the transformation coefficient η is computed by using equation (8).

The plotting of the result using this method consists of the following steps:

(1) As shown by Scheme 4, the plotting η, N informs us of the variation of the physical parameter considered in the lunar band observed. With respect to this, the curve of Scheme 4 apparently shows nothing new when we compare its shape with curves obtained from other procedures. As we can see by the following example, however, this conclusion is not realistic.

In Table 1, $P_{25} = P_o = 0.18$. If we assume that the number 0.18 represents a white color, for instance, the equality $P_{25} = P_o$ would mean that both areas have the same color. But in Scheme 4 the ordinate of P_{25} is too high. In terms of successive transformations, this would mean that both areas effectively have the same color with the difference being; for example, that one of them is a flat white surface while the other is a granular white surface.

(2) As shown by Figure 1 (see Appendix), plotting η, N, and slope of P_o gives a direct answer to the question about safety previously referenced. If we assume, for example, that point P_o of Surveyor corresponds to an area which except for being too hot, is in all respects satisfactory for landing, other points placed on its slope would have the same nature as P_o. The only difference being that the temperature varies as a function of latitude.

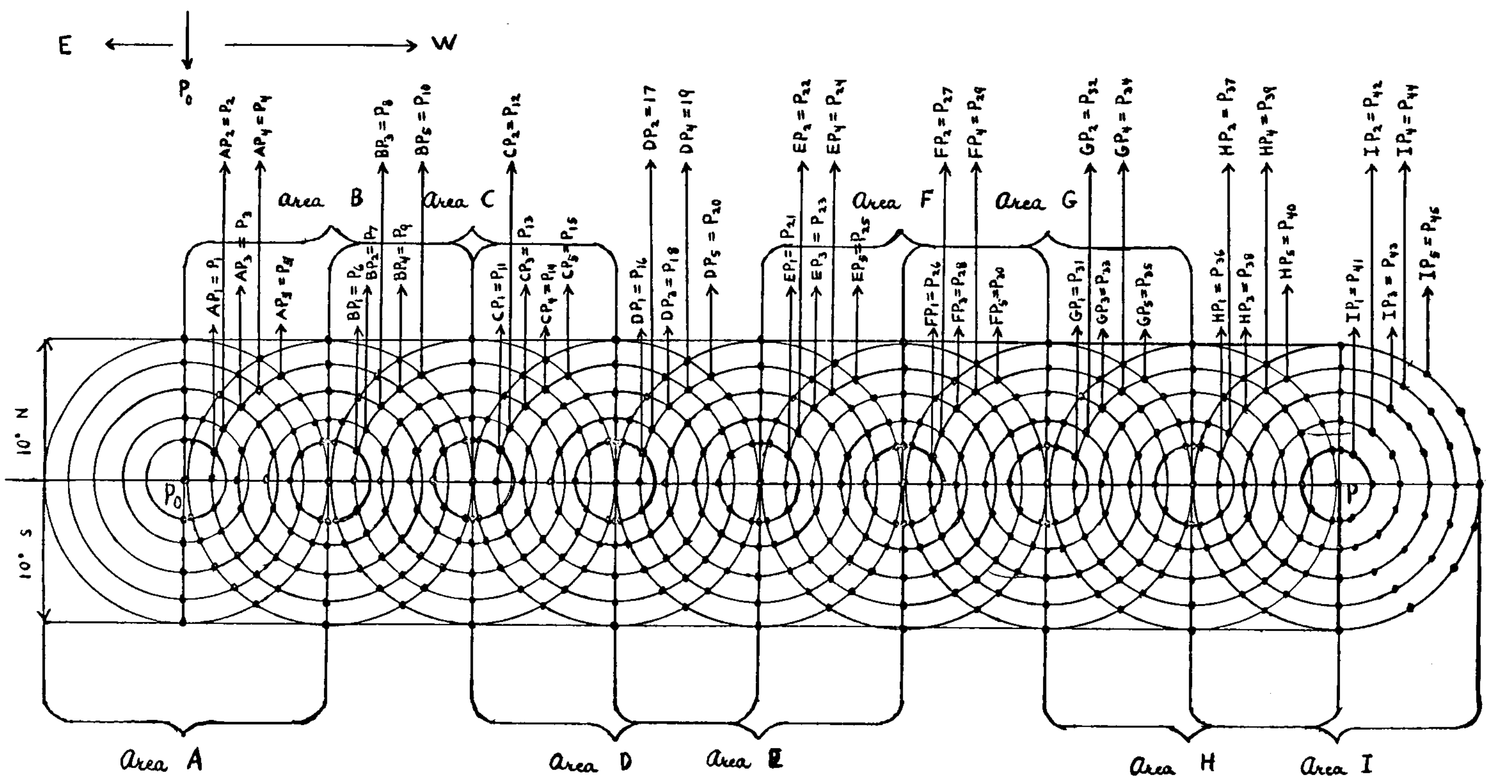
In our examples, Figure 1 shows that P_{22} is the safest point of those whose data were studied within the successive transformations method. The points P_1, P_2, P_3, and P_4, but especially P_1, appear also as good as P_o, but this is not surprising because they are the closest points to P_o.

There now remains only a final question: "How to define exactly those surface properties identified as the safest for landing a manned spacecraft on the moon?" After identification, the area will be observed again to establish the variation of η with the different phases of the moon. The systematic errors will be eliminated by observing that area at least three times, with each observation corresponding to a different lunation. The mean of these three observations will be used for plotting η, phases of the moon and to obtain, in this way, the new photometric function given by the successive transformations method.

This new photometric function will give us the precise information desired about the physical properties of the area mentioned.

Concerning the observations themselves, a correction will be introduced with regard to the extinction coefficient of our atmosphere. For this, an "Air Mass Table" will be prepared by adopting the least square solutions in order to obtain the extinction coefficient pertaining to the dates of observations, geographical coordinates of the observatory and instrument used. This will be the part of this research for the Computer Program.

APPENDIX



SCHEME I

P_o = Landing site of Surveyor

P = Landing site selected for a Manned Spacecraft

$P_1, P_2, P_3, \ldots, P_{45}$ = Examples of successive transformations for observational data

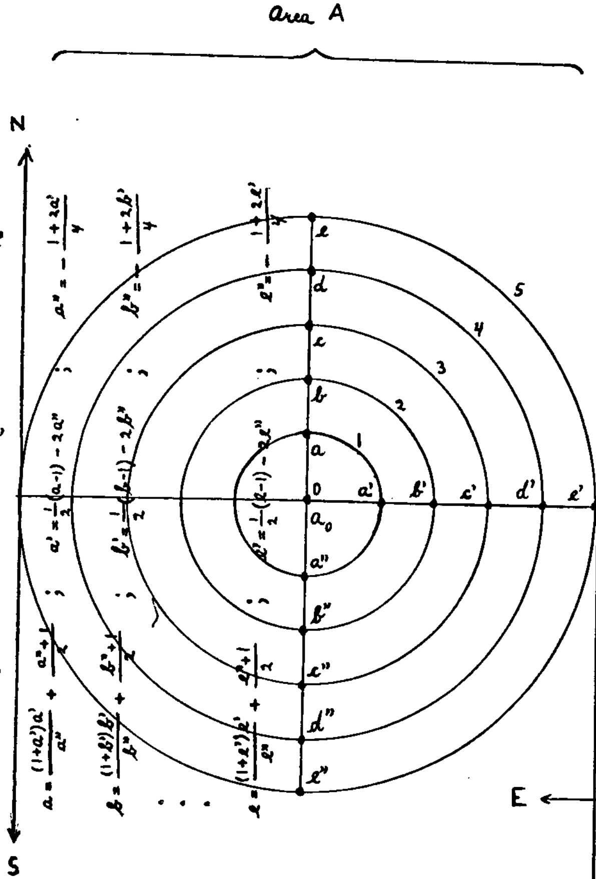
$$a_0 = \frac{(\mathscr{E}_i)_0}{(\mathscr{E}_i)_0} = 1$$

$$\frac{(\mathscr{E}_i)_0}{(\mathscr{E}_i)_1} = a \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}'_i)_1} = a' \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}''_i)_1} = a''$$

$$\frac{(\mathscr{E}_i)_0}{(\mathscr{E}_i)_2} = b \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}'_i)_2} = b' \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}''_i)_2} = b''$$

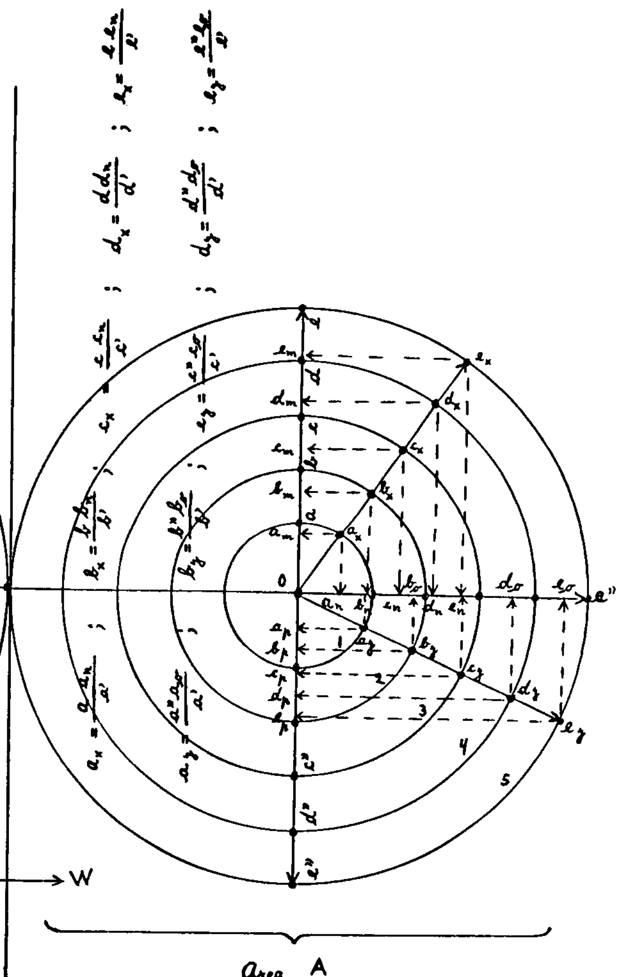
$$\vdots$$

$$\frac{(\mathscr{E}_i)_0}{(\mathscr{E}_i)_5} = e \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}'_i)_5} = e' \qquad \frac{(\mathscr{E}_i)_0}{(\mathscr{E}''_i)_5} = e''$$



SCHEME 2

Enlargement of the site where Surveyor has landed



SCHEME 3

Definition of the points $(a_x, a_y), (b_x, b_y), \ldots, (e_x, e_y)$ with respect to $(a, a', a''), (b, b', b''), \ldots, (e, e', e'')$.

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TABLE I

Examples of successive transformations for some points above P_o = 0.18 in the SCHEME 1

Area A in the SCHEME 1

P	ΔP	η	N
$P_1 = P_1 = 0.23$	0.01	0.77	2
$P_2 = P_2 = 0.21$	0.01	0.77	3
$P_3 = P_3 = 0.14$	0	1.01	4
$P_4 = P_4 = 0.09$	-0.01	1.31	5
$P_5 = P_5 = 0.17$	0.01	0.63	6

Area B in the SCHEME 1

P	ΔP	η	N
$P_1 = P_6 = 0.36$	0	0.30	7
$P_2 = P_7 = 0.27$	0	0.37	8
$P_3 = P_8 = 0.11$	-0.01	0.88	9
$P_4 = P_9 = 0.47$	-0.01	0.17	10
$P_5 = P_{10} = 0.06$	0	1.16	11

Area C in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{11} = 0.31$	0	0.19	12
$P_2 = P_{12} = 0.55$	0	0.09	13
$P_3 = P_{13} = 0.13$	0	0.30	14
$P_4 = P_{14} = 0.03$	0	1.00	15
$P_5 = P_{16} = 0.16$	0	0.12	16

Area D in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{16} = 0.69$	0.01	0.01	17
$P_2 = P_{17} = 0.23$	0.01	0.60	18
$P_3 = P_{18} = 0.14$	-0.02	1.14	19
$P_4 = P_{19} = 0.08$	0	2.25	20
$P_5 = P_{20} = 0.60$	0.01	0.33	21

Area E in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{21} = 0.92$	-0.02	0.24	22
$P_2 = P_{22} = 0.29$	0.01	0.82	23
$P_3 = P_{23} = 0.65$	0	0.40	24
$P_4 = P_{24} = 0.04$	0	7.00	25
$P_5 = P_{25} = 0.18$	0	1.66	26

Area F in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{26} = 0.22$	0	1.45	27
$P_2 = P_{27} = 0.86$	0.01	0.39	28
$P_3 = P_{28} = 0.55$	0.01	0.65	29
$P_4 = P_{29} = 0.73$	0	0.52	30
$P_5 = P_{30} = 0.13$	0	3.07	31

Area G in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{31} = 0.25$	0	1.68	32
$P_2 = P_{32} = 0.09$	0.01	4.88	33
$P_3 = P_{33} = 0.71$	0.01	0.67	34
$P_4 = P_{34} = 0.43$	0	1.16	35
$P_5 = P_{35} = 0.19$	0	2.73	36

Area H in the SCHEME 1

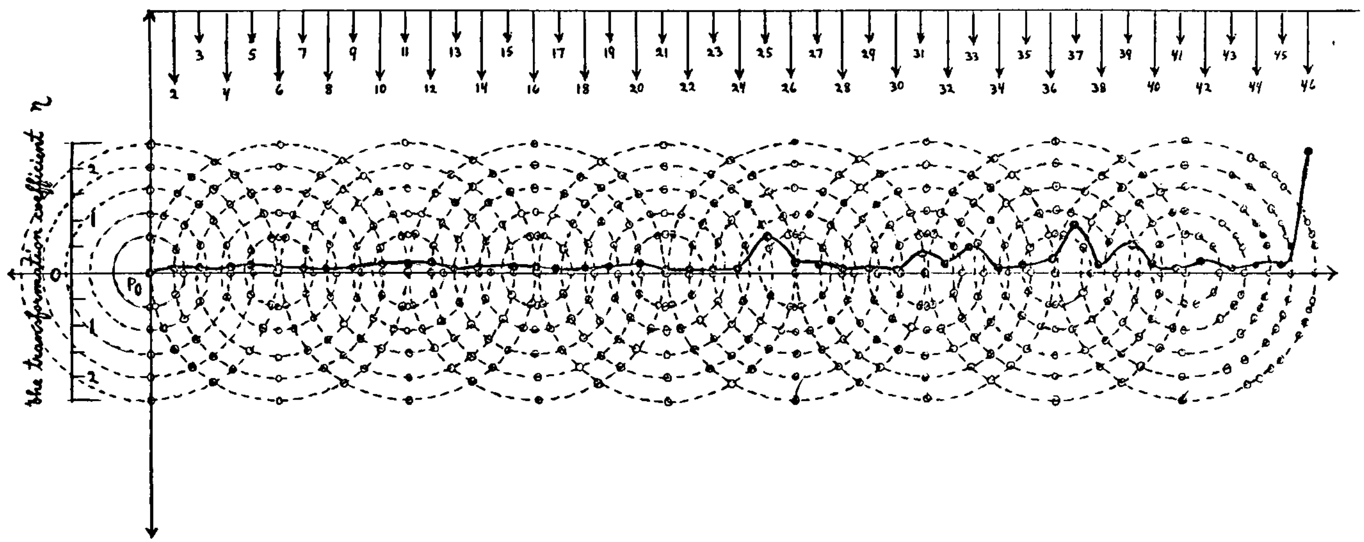
P	ΔP	η	N
$P_1 = P_{36} = 0.06$	0	9.00	37
$P_2 = P_{37} = 0.39$	0.01	1.43	38
$P_3 = P_{38} = 0.11$	0	5.09	39
$P_4 = P_{39} = 0.37$	0.01	1.56	40
$P_5 = P_{40} = 0.87$	0.01	0.68	41

Area I in the SCHEME 1

P	ΔP	η	N
$P_1 = P_{41} = 0.27$	0	2.29	42
$P_2 = P_{42} = 0.79$	0	0.81	43
$P_3 = P_{43} = 0.54$	0.01	1.22	44
$P_4 = P_{44} = 0.36$	0.01	1.88	45
$P_5 = P_{45} = 0.03$	0	23.33	46

- - - Circular areas displayed in SCHEME I

..... Scan points resulting from successive transformations



SCHEME 4

Illustration of a typical scan showing the successive transformation of observational data and related transformation coefficients

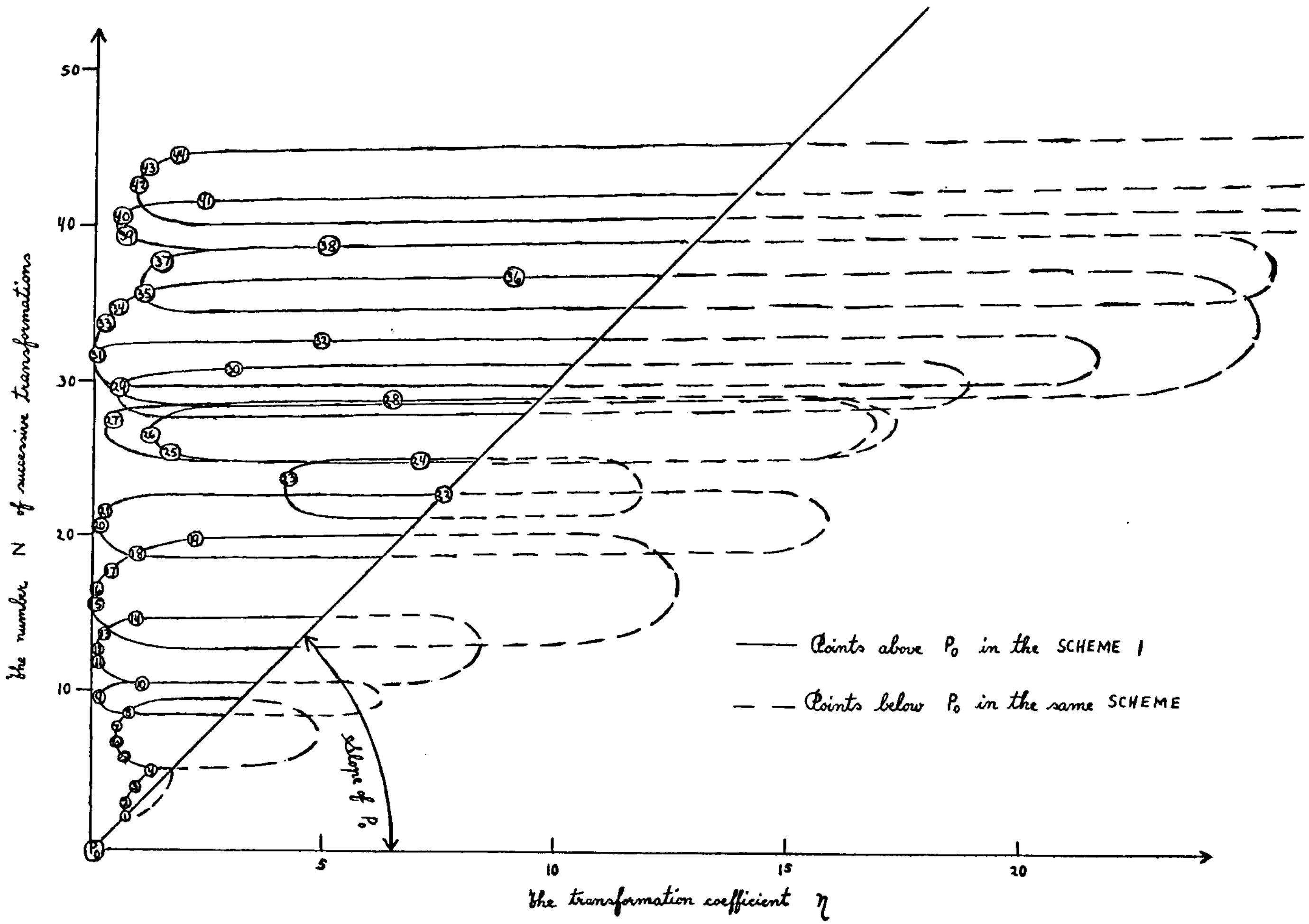

50
40
30
20
10
5
10
15
20
the number N of successive transformations
the transformation coefficient η
Slope of P_0
Points above P_0 in the SCHEME I
Points below P_0 in the same SCHEME

FIGURE 1.

EXAMPLES NO. 1, NO. 2 AND NO. 3

Let, in the region A, $P_0 = 0.18$ the data concerning the point where Surveyor landed. Let $P_1 = 0.23$, $P_2 = 0.21$ and $P_3 = 0.14$ the corresponding data of those points situated at, respectively, the 1st, 2nd, and 3rd circles.

What are the transformation coefficients α, β and γ with respect to P_0?

What is the efficiency on the result obtained, or error committed in the successive transformations, which must be considered when analyzing data to select an area on the Moon?

Example n° 1

$$P_0 = 0.18 \longrightarrow N=0 \longleftarrow P_1 = 0.23$$

$$\frac{P_1}{P_0} = 1.29 \longrightarrow N=1 \longleftarrow \frac{P_1}{P_1} = 1.00$$

$$\overbrace{\frac{P_1}{P_1}\Big/\frac{P_0}{P}}^{N=2} = \alpha = 0.77$$

So $P_0 = \alpha P_1 + \frac{N}{100} = (0.77)(0.23) + 0.02 = 0.19$ and the error is $\left(\alpha P_1 + \frac{N}{100}\right) - P_0 = 0.19 - 0.18 = 0.01$

Example n° 2

$$P_0 = 0.18 \longrightarrow N=0 \longleftarrow P_1 = 0.23 \longrightarrow N=0 \longleftarrow P_2 = 0.21$$

$$\frac{P_0}{P_2} = 0.85 \longrightarrow N=1 \longleftarrow \frac{P_1}{P_2} = 1.04 \longrightarrow N=1 \longleftarrow \frac{P_2}{P_2} = 1.00$$

$$\frac{P_1}{P_2}\Big/\frac{P_0}{P_2} = 1.22 \rightarrow N=2 \leftarrow \frac{P_2}{P_2}\Big/\frac{P_1}{P_2} = 0.95$$

$$\overbrace{\frac{P_2}{P_2}\Big/\frac{P_1}{P_2}\bigg/\frac{P_1}{P_2}\Big/\frac{P_0}{P_2}}^{N=3} = \beta = 0.77$$

So $P_0 = \beta P_2 + \frac{N}{100} = (0.77)(0.21) + 0.03 = 0.19$ and the error is $\left(\beta P_2 + \frac{N}{100}\right) - P_0 = 0.19 - 0.18 = 0.01$

Example n° 3

$$P_0 = 0.18 \longrightarrow N=0 \longleftarrow P_1 = 0.23 \longrightarrow N=0 \longleftarrow P_2 = 0.21 \longrightarrow N=0 \longleftarrow P_3 = 0.14$$

$$\frac{P_0}{P_3} = 1.28 \longrightarrow N=1 \longleftarrow \frac{P_1}{P_3} = 1.64 \longrightarrow N=1 \longleftarrow \frac{P_2}{P_3} = 1.50 \longrightarrow N=1 \longleftarrow \frac{P_3}{P_3} = 1.00$$

$$\frac{P_1}{P_3}\Big/\frac{P_0}{P_3} = 1.28 \longrightarrow N=2 \longleftarrow \frac{P_2}{P_3}\Big/\frac{P_1}{P_3} = 0.91 \longrightarrow N=2 \longleftarrow \frac{P_3}{P_3}\Big/\frac{P_2}{P_3} = 0.66$$

$$\frac{P_2}{P_3}\Big/\frac{P_1}{P_3}\bigg/\frac{P_1}{P_3}\Big/\frac{P_0}{P_3} = 0.71 \longrightarrow N=3 \longleftarrow \frac{P_3}{P_3}\Big/\frac{P_2}{P_3}\bigg/\frac{P_2}{P_3}\Big/\frac{P_1}{P_3} = 0.72$$

$$\overbrace{\frac{P_3}{P_3}\Big/\frac{P_2}{P_3}\bigg/\frac{P_2}{P_3}\Big/\frac{P_1}{P_3}\Bigg/\frac{P_2}{P_3}\Big/\frac{P_1}{P_3}\bigg/\frac{P_1}{P_3}\Big/\frac{P_0}{P_3}}^{N=4} = \gamma = 1.01$$

So $P_0 = \gamma P_3 + \frac{N}{100} = (1.01)(0.14) + \frac{4}{100} = 0.14 + 0.04 = 0.18$

and the error is $\left(\gamma P_3 + \frac{N}{100}\right) - P_0 = 0.18 - 0.18 = 0$

EXAMPLE NO. 4

Let, in th[illegible] region A, P_o = 0.18 the data concerning the point wh[illegible]re Sur[illegible]yor la[illegible]d. [illegible]et P = 0.23, P_2 = 0.21, P_3 = 0.14 and P_4 = 0.09 the corresponding data of thos[illegible] points [illegible]tuated a[illegible], respectivel[illegible], the 1st, 2nd, 3rd and 4th circles.

What is th[illegible] transformation coefficient, δ, of P_4 with respect to P_o?

What is th[illegible] efficiency of the result obtain[illegible], or error committed in th[illegible] succes[illegible] transformations, which must be considered when analyzing data to select an area o[illegible] the Moo[illegible]

$$P_0 = 0.18 \longrightarrow N=0 \longleftarrow P_1 = 0.23 \longrightarrow N=0 \longleftarrow P_2 = 0.21 \longrightarrow N=0 \longleftarrow P_3 = 0.14 \longrightarrow N=0 \longleftarrow P_4 = 0.09$$

$$\frac{P_0}{P_4} = 2.00 \longrightarrow N=1 \longleftarrow \frac{P_1}{P_4} = 2.55 \rightarrow N=1 \leftarrow \frac{P_2}{P_4} = 2.33 \longrightarrow N=1 \longleftarrow \frac{P_3}{P_4} = 1.55 \rightarrow N=1 \longleftarrow \frac{P_4}{P_4} = 1.00$$

$$\frac{P_1}{P_4}\Big/\frac{P_0}{P_4} = 1.27 \rightarrow N=2 \leftarrow \frac{P_2}{P_4}\Big/\frac{P_1}{P_4} = 0.91 \rightarrow N=2 \leftarrow \frac{P_3}{P_4}\Big/\frac{P_2}{P_4} = 0.66 \rightarrow N=2 \leftarrow \frac{P_4}{P_4}\Big/\frac{P_3}{P_4} = 0.64$$

$$\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\Bigg/\left(\frac{P_1}{P_4}\right)\Big/\left(\frac{P_0}{P_4}\right) = 0.71 \rightarrow 3 \leftarrow \left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right) = 0.72 \rightarrow 3 \leftarrow \left(\frac{P_4}{P_4}\right)\Big/\left(\frac{P_3}{P_4}\right)\Bigg/\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right) = 0.96$$

$$\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\Bigg/\!\!\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\Bigg/\left(\frac{P_1}{P_4}\right)\Big/\left(\frac{P_0}{P_4}\right) = 1.01 \longrightarrow N=4 \longleftarrow \left(\frac{P_4}{P_4}\right)\Big/\left(\frac{P_3}{P_4}\right)\Bigg/\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\!\!\Bigg/\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right) = 1.33$$

$$\overbrace{\left[\left(\frac{P_4}{P_4}\right)\Big/\left(\frac{P_3}{P_4}\right)\Bigg/\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\!\!\Bigg/\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\right]\Bigg/\left[\left(\frac{P_3}{P_4}\right)\Big/\left(\frac{P_2}{P_4}\right)\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\Bigg/\!\!\Bigg/\left(\frac{P_2}{P_4}\right)\Big/\left(\frac{P_1}{P_4}\right)\Bigg/\left(\frac{P_1}{P_4}\right)\Big/\left(\frac{P_0}{P_4}\right)\right]}^{N=5} = \delta = 1.31$$

EXPLANATION

Beginning from P_o of the point where Surve[illegible]or landed, and continuing t[illegible] the poi[illegible] P_4 of th[illegible] [illegible]th circl[illegible] of [illegible]he region A, we need to do in th[illegible] case a number N of succe[illegible]ve tra[illegible]ormations, [illegible]qual to [illegible] to obtain th[illegible] corresponding co[illegible]ficient of tra[illegible]sformations [illegible] P_4 with respe[illegible] to P_o.

[illegible]o I_o = [illegible] $P_4 + \frac{[illegible]}{100}$ = (1.31)(0.09) + $\frac{5}{100}$ = 0.[illegible]7,

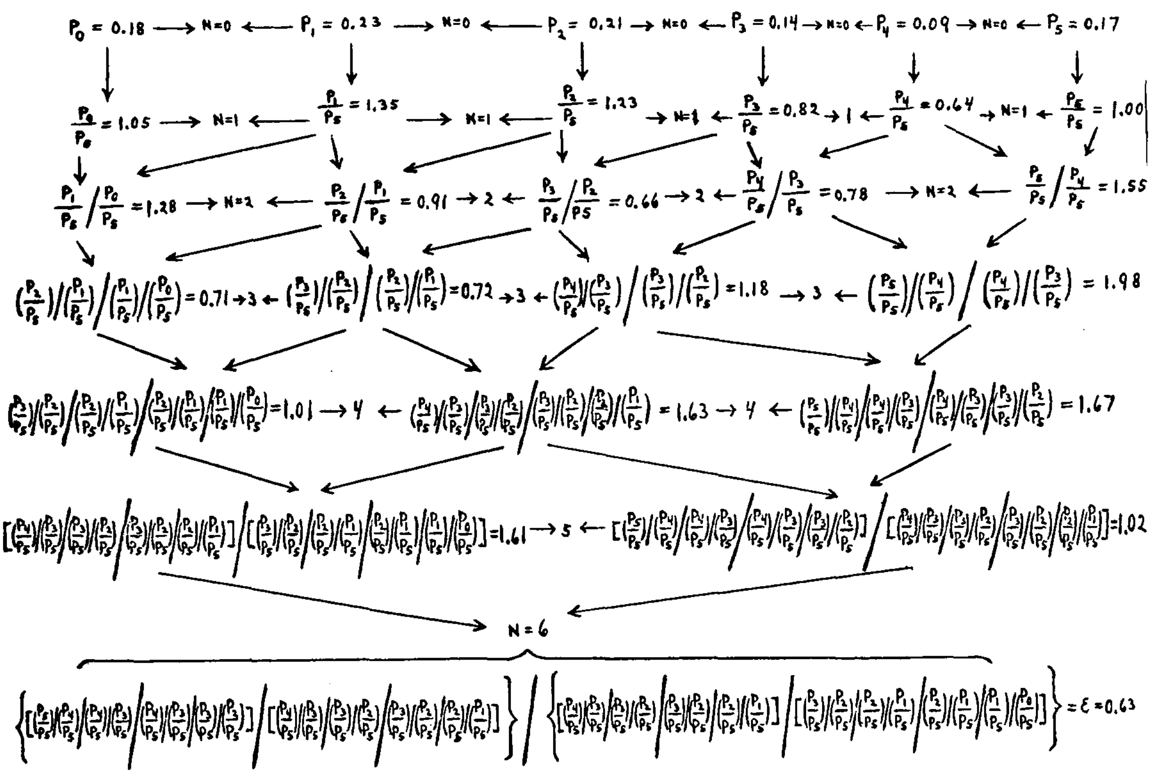
and the err[illegible]r is $(\delta P_4 + \frac{[illegible]}{100}) - I_o$ = 0.[illegible] - 0.[illegible] [illegible]

EXAMPLE NO. 5

Let, in the region A, P_o = 0.18 the data concerning the point where Surveyor landed. Let P_1 = 0.23, P_2 - 0.21, P_3 = 0.14, P_4 = 0.09 and P_5 = 0.17 the corresponding data of those points situated at, respectively, the 1st, 2nd, 3rd, 4th and 5th circles.

What is the transformation coefficient ϵ, of P_5 with respect to P_o?

What is the efficiency on the result obtained, or error committed in the successive transformations, which must be considered when analyzing data to select an area on the Moon?



EXPLANATION

Beginning from P_o of the point where Surveyor landed, and continuing to the point P_5 of the 5th circle of the region A, we need to do in this case a number N of successive transformations, ... to obtain the corresponding coefficient of transformation ... with respect to P_o.

So ... $= \epsilon P_5 + \frac{N}{100}$ = (0.63)(0.17) + $\frac{6}{100}$ = 0.19,

and the error is $(\epsilon P_5 + \frac{N}{100}) - P_o$ = 0. ... - 0. ... ...